

## CORRIGENDA

Generalized expressions for secondary vorticity using intrinsic coordinates

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*Journal of Fluid Mechanics*, vol. 59 (1973), pp. 97–115

Our attention has been drawn by James (1987; 1980 private communication) and M. B. Okan & D. G. Gregory Smith (1990 private communication) to some errors in our paper on generalized equations for secondary vorticity. These arise from the use of the following expressions for the differential coefficients of the unit vector along the streamline:

$$\frac{\partial s}{\partial n} = \frac{\mathbf{n}}{a_n} \frac{\partial a_n}{\partial s}, \quad \frac{\partial s}{\partial b} = \frac{\mathbf{b}}{a_b} \frac{\partial a_b}{\partial s}.$$

James and Okan & Gregory Smith correctly show that these expressions are valid only when the streamwise vorticity is zero.

James provides more generalized expressions for  $\partial s/\partial n$ ,  $\partial s/\partial b$ , but was unable to relate this to the flow field. James and Okan & Gregory Smith suggest representing these terms by

$$\frac{\partial s}{\partial n} = \alpha_1 \mathbf{n} + \beta_1 \mathbf{b}, \quad \frac{\partial s}{\partial b} = \alpha_2 \mathbf{n} + \beta_2 \mathbf{b}.$$

More general expressions for  $\partial s/\partial n$ ,  $\partial s/\partial b$  can be derived for the case of  $\omega_s \neq 0$ . Figure 1 shows a ‘nest’ of streamlines which rotate about the streamline direction  $s$ , the  $n$ - and  $b$ -axes rotating by amounts  $\delta\psi_n$  and  $\delta\psi_b$  in a distance  $\delta s$ . The angles of divergence of the streamlines (in the  $n$ - and  $b$ -directions) are  $\delta\phi_n = \partial a_n/\partial s$  and

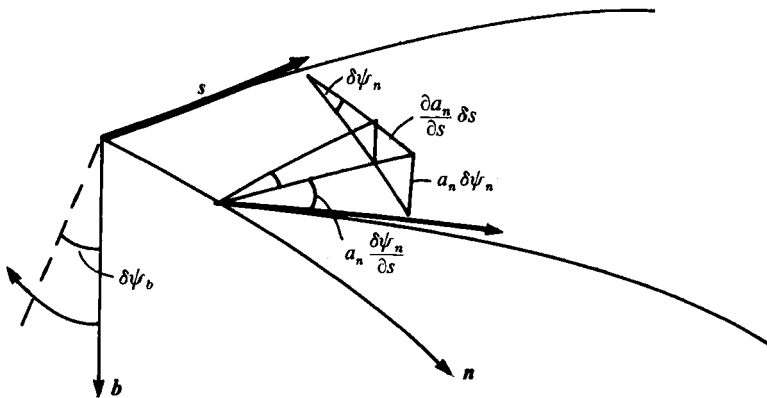


FIGURE 1.

$\delta\phi_b = \partial a_b / \partial s$ . The difference between the streamwise vectors at  $n = 0$  and  $n = a_n$  is then

$$\delta\mathbf{s} = \mathbf{n}\delta\phi_n + \mathbf{b}\left(\frac{a_n\delta\psi_n}{\delta s}\right),$$

so that

$$\frac{\partial\mathbf{s}}{\partial n} = \alpha_1\mathbf{n} + \beta_1\mathbf{b},$$

where

$$\alpha_1 = \frac{1}{a_n} \frac{\partial a_n}{\partial s}, \quad \beta_1 = \frac{\partial\psi_n}{\partial s}.$$

Similarly

$$\frac{\partial\mathbf{s}}{\partial b} = \alpha_2\mathbf{n} + \beta_2\mathbf{b},$$

where

$$\alpha_2 = -\frac{\partial\psi_b}{\partial s}, \quad \beta_2 = \frac{1}{a_b} \frac{\partial a_b}{\partial s}.$$

Following Shapiro (1953, p. 270), the streamwise vorticity ( $\omega_s$ ) is twice the fluid rotation, so that

$$\frac{1}{2}\omega_s = \frac{1}{2}\left(\frac{\partial\psi_n}{\partial t} + \frac{\partial\psi_b}{\partial t}\right),$$

$$\omega_s = q\left(\frac{\partial\psi_n}{\partial s} + \frac{\partial\psi_b}{\partial s}\right) = q(\beta_1 - \alpha_2).$$

When the modified expressions for  $\partial\mathbf{s}/\partial n$ ,  $\partial\mathbf{s}/\partial b$  are used in the analysis developed in our original paper, the various equations for the growth of streamwise (secondary) vorticity are unchanged. However, the equations for the growth of normal vorticity ( $\omega_n$ ) have to be corrected. For example, equation (18) becomes

$$\rho q \frac{\partial}{\partial s} \left( \frac{\omega_n}{\rho} \right) = q\omega_b \left[ \frac{1}{\tau} - \frac{\partial\psi_b}{\partial s} \right] + \frac{\omega_n q}{a_n} \frac{\partial a_n}{\partial s} + \frac{1}{\rho^2} \left[ \frac{\partial p}{\partial s} \frac{\partial \rho}{\partial b} - \frac{\partial p}{\partial b} \frac{\partial \rho}{\partial s} \right]. \quad (1)$$

An additional term ( $-q\omega_b(\partial\psi_b/\partial s)$ ) is now included on the right-hand side, but like the term ( $q\omega_b/\tau$ ) can be neglected in most situations because  $\omega_b$  is normally small. (The major effect is the 'stretching' of the vorticity due to the change of  $a_n$  with  $s$ .) The extra terms should also be added to equations (20) and (38). Equations (67), (68) and (70) will have an additional term [ $-W\omega_b(\partial\psi_b/\partial s')$ ] on the right-hand side. Here the prime refers to the relative coordinate system.

James has also shown that this omission does not change the transport equation for the streamwise vorticity. His expression for normal vorticity includes the term  $\partial\mathbf{s}/\partial b$ , which can now be replaced by the expression given in this corrigendum. His expression (James 1987, equation 23) would then be identical to equation (1) above. James also provides a transport equation for  $\omega_b$ .

Some additional corrections brought to our attention by readers are as follows.

- (i) Equation (55), p. 108:  $\nabla'\zeta$  should read  $\nabla' \times \zeta$ .
- (ii) Equation (58), p. 109: fourth term should read

$$\frac{2\Omega_{s'} \partial \rho}{\rho^2 W \partial s'}$$

- (iii) The equation above (67): the term  $\partial a_{b'}/\partial b'$  should read  $\partial a_{b'}/\partial s'$ .
- (iv) Delete the term  $2\Omega_{n'} \partial W/\partial s'$  in (68).

#### REFERENCES

- JAMES, P. W. 1987 *Proc. Inst. Mech. Engrs* **201**, 413.  
 SHAPIRO, A. 1953 *Dynamics and Thermodynamics of Compressible Fluid Flow*. Wiley.

Surface wave and thermocapillary instabilities in a liquid film flow

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*Journal of Fluid Mechanics*, vol. 223 (1991), pp. 25–45

$\Delta T$  was mistakenly defined as the difference between the wall and the ambient temperatures, i.e.

$$\Delta T = T_w - T_0.$$

The correct definition of  $\Delta T$  is the difference between the wall and the free-surface temperatures, i.e.

$$\Delta T = T_w - T(0) = (T_w - T_0) \frac{Bi}{1 + Bi}.$$